THE PROBABILISTIC MODEL OF THE DYNAMIC OF THE CABLES UNDER SHORT-CIRCUIT CURRENT.

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Abstract
The algorithm of the numerical solution of the cable's differential equation is modified in a way that permits to improve already published results. The algorithm is derived by linearization of the nonlinear equation of the cable motion. Unlike the known algorithms, which are used for relatively highly extensible cables, this one is efficient for the numerical solution of slightly extensible and non-extensible cables. The comparative results of the calculation according to the two methods are presented here. The probabilistic approach to the short-circuit current is given as well. This approach is similar to the design criterion for random wind loads with return periods of 50, 100, 150 etc years.

Keywords: Probability, Short-circuit current, Cable, Substation, Finite difference, Nonlinear structure analyses.

1. Introduction.

The short-circuit current is one of the most important factors influencing the design of the electrical substations. The short-circuit current is a wide-spread fault of an electrical system, when the electrical current is drastically increased, as compared with the operation conditions and dropped after a short time. This current induces significant transient electromagnetic forces in a cable (conductor), as well as its substantial deflection.

The first purpose of this paper is intended to refine the accuracy of the deterministic calculation methods. The second purpose is to choose the design criterion similar to that adopted for random wind loads acting on electrical transmission lines.

For the purpose of the paper's study, an electrical transmission line is represented as a uniform, perfectly flexible (without bending and torque stiffness), extensible cable. The cable is supported at its two ends and subjected to transverse load due to gravity, wind and electromagnetic forces.

The method such as that used in [6, 7] for obtaining the numerical solution of the cables dynamic equations under short-circuit current gives good results for the first maximum of the tension that occurs at the time close to the drop of the external electromagnetic force. The practical rules [1,2] for the calculation of the effect of the short-circuit current are based on this method. However an accumulation of calculation errors is inherent in this method. These errors lead to considerable inaccuracy in the evaluation of the cable tension, especially for relatively slightly stretched and heavy cables under relatively low short-circuit current, when the maximum of the tension is achieved after an relatively extended period of time. The method is not efficient for slightly extensible cables and it is quite unusable for non-extensible cables.

The algorithm derived in this paper is free from the above mentioned limitations.

The distribution of the expected short-circuit current in the system may be found by Monte-Carlo simulations [3]. Different aspects of the probabilistic approach for calculation of the effects of the short-circuit current are known from [4, 5].

The probabilistic approach to the short-circuit current, which is based on the algorithm above and which is derived in this paper enable us to calculate the mean time of appearance of a tension greater than some specified value. This approach is similar to the design criterion for random wind loads with the return periods of 50, 100, 150 ect years.

2. Equations of motion.

We write the nondimentional dynamic equations of the cable motion in the following form:

$$\lambda\frac{\partial^2 X_i}{\partial t^2}-\frac{\partial}{\partial S}\left(T\frac{\partial X_i}{\partial S}\right)-F_i=0; \qquad (i=1,2,3)$$

$$\sum_{i=1}^{3}\left(\frac{\partial X_i}{\partial S}\right)^2=1; \qquad (1)$$

$$\frac{\partial S_0}{\partial S}=\frac{1}{1+\alpha T},$$

where X_i, T, F_i, t, S_0, S are the nondimentional: “Cartesian coordinates“, “cable tension“, “external forces“, “time“, “arc undeformed and deformed cable coordinates“, respectively (the term nondimentional and the quotes are omitted below in the text for these values);

$$X_i=\frac{X_{id}}{\ell};\quad S=\frac{S_d}{\ell};\quad S_0=\frac{S_{d0}}{\ell};\quad T=\frac{T_d}{\beta mg\ell};\quad F_i=\frac{F_{di}}{\beta mg};\quad t=t_d\omega.$$

T_d is the real cable tension; X_{id} (i=1, 2, 3) are the real Cartesian coordinates of a cable point, F_{di} are the real projections of the external force on Cartesian axis, S_{d0}, S_d are the real arc (Euler) undeformed and deformed coordinate along cable mode (before and after loading) respectively, t_d is the real time, β is the nondimentional coefficient, that equates the order of the tension with the rest of variables. Its value is between 100 to 10000, A_s is the area of the cable cross section, E is the modulus of elasticity of the cable's material, m is the mass of the cable's unit of length, g is the gravity constant, ℓ is the undeformed cable length between two ends (before loading),

ω is the current frequency of the electrical system, α, λ are nondimensional constants;

$$\lambda=\frac{\ell\omega^2}{\beta g};\quad \alpha=\frac{\beta mg\ell}{A_s E}.$$

There is a set of five equations (1) of cable motion. The first three equations concern the motion of a (spatial) point on the current curvilinear coordinates *S* along the cable. The fourth one relates the Cartesian coordinates of spatial points on the cable with the current curvilinear coordinates *S* along the cable. The fifth equation relates the current curvilinear coordinates *S* to the undeformed curvilinear coordinates S_0. Essentially, the current curvilinear coordinates *S* measures the current length of the cable.

In term of

$$\frac{d}{\partial S}=\frac{d}{\partial S_0}\frac{1}{(1+\alpha T)} \tag{2}$$

we obtain the equations as related to the independent variable S_0

$$\lambda(1+\alpha T)\frac{\partial^2 X_i}{\partial t^2}-\frac{\partial}{\partial S_0}(\frac{T}{1+\alpha T}\frac{\partial X_i}{\partial S_0})-F_i(1+\alpha T)=0\,;$$

$$\sum_{i=1}^{3}(\frac{\partial X_i}{\partial S_0})^2=(1+\alpha T)^2\,; \tag{3}$$

$$\frac{\partial S}{\partial S_0}=1+\alpha T\,.$$

In the case, when the ends of the cable are fixed points, the boundary conditions are

$$X_i(0,t)=0;\quad X_i(\ell,t)=0\,. \tag{4}$$

The initial conditions are

$$X_i(S_0,0)=\varphi_i(S_0);\quad \frac{\partial X_i(S_0,0)}{\partial t}=0\,. \tag{5}$$

We assume that the initial speed of the cable is zero and the initial static profile $\varphi_i(S_0)$ is known and may be calculated by one of the static methods (for instance [6]). If before the short-circuit current the cable is subjected only to gravity force, the initial static profile is a catenary line.

We obtain the linear equations regarding the small increments x_i, h, s of the coordinates, tension and arc length of the cable for each small increment of the time Δ_t, by performing in Eqs.(3) the replacement:

$$X_i=\widetilde{X}_i+x_i,\quad T=\widetilde{T}+h,\quad S=\widetilde{S}+s,\quad F_i=\widetilde{F}_i+f_i\,, \tag{6}$$

where $\widetilde{X}_i, \widetilde{T}, \widetilde{S}, \widetilde{F}_i$ are the values corresponding to the arbitrary time *t,* X_i, T, S, F_i are the values corresponding to the time $t+\Delta_t$.

We confine in equations only the values of the same order as the increments x_i, h, s, f_i, in terms of

$$\frac{d}{\partial S_0}(\frac{\widetilde{T}}{1+\alpha\,\widetilde{T}})=\frac{\partial\widetilde{T}}{\partial S_0}\frac{1}{(1+\alpha\,\widetilde{T})^2}\;;$$

$$\frac{1}{1+\alpha\,\widetilde{T}}=1+\alpha\,\widetilde{T}+(\alpha\,\widetilde{T})^2+\ldots\;; \tag{7}$$

$$\frac{1}{(1+\alpha\,\widetilde{T})^2}=1+2\alpha\,\widetilde{T}+(2\alpha\,\widetilde{T})^2+\ldots,$$

considering that $\alpha\,\widetilde{T} << 1$, after manipulations we achieve:

$$\lambda\frac{\partial^2 x_i}{\partial t^2}-\widetilde{T}\frac{\partial^2 x_i}{\partial S_0{}^2}-\frac{\partial^2\widetilde{X}_i}{\partial S_0{}^2}h-\frac{\partial\widetilde{T}}{\partial S_0}\frac{\partial x_i}{\partial S_0}-\frac{\partial\widetilde{X}_i}{\partial S_0}\frac{\partial h}{\partial S_0}+\lambda\frac{\partial^2\widetilde{X}_i}{\partial t^2}\alpha h-\widetilde{F}_i\alpha h-f_i=0$$

$$\sum_{i=1}^{3}(\frac{\partial\widetilde{X}_i}{\partial S_0}\frac{\partial x_i}{\partial S_0})-2\alpha\,h=0\,; \tag{8}$$

$$\frac{\partial s}{\partial S_0}=\alpha\,h\qquad (i=1,\,2,\,3).$$

3. External forces.

An electric conductor (cable) carrying current $I_1(t_d)$, in a magnetic field due to another conductor carrying current $I_2(t_d)$, undergoes an electromagnetic force F_d which is acting on a unit length of the cable, and that is related to the currents $I_1(t_d)$ and $I_2(t_d)$ by:

$$F_d = \frac{\mu_0}{2\pi} \frac{I_1(t_d)\ I_2(t_d)}{a}, \qquad (9)$$

where μ_0 is the electromagnetic constant, a is the distance between the conductors.

According to the recommendations of CIGRE [3] the short-circuit current is described by formula:

$$I(t_d) = I_a[\sin(\omega\, t_d + \xi_u - \gamma_z) + \sin(\gamma_z - \xi_u)e^{\frac{t_d}{\tau}}], \qquad (10)$$

where I_a is the amplitude of the short-circuit current (at time zero), ξ_u, γ_z, τ are constants

4. Numerical equations.

We discretize the Eqs. (8) to obtain the finite difference equations. The cable motion is approximated by N cable point, $(N+1)$ cable linear elements of constant undeformed length Δ_{S0} and M constant steps of time Δ_t.

$$x_{i,j}^{k+1} - 2x_{i,j}^{k} + x_{i,j}^{k-1} - \delta_1[\widetilde{T}_j^k(x_{i,j+1}^{k+1} - 2x_{i,j}^{k+1} + x_{i,j-1}^{k+1}) - (\widetilde{T}_j^k - \widetilde{T}_{j-1}^k)(x_{i,j}^{k+1} - x_{i,j-1}^{k+1}) - \qquad (11)$$

$$-(\widetilde{X}_{i,j+1}^k - 2\widetilde{X}_{i,j}^k + \widetilde{X}_{i,j-1}^k)h_j^{k+1} - (h_j^{k+1} - h_{j-1}^{k+1})(\widetilde{X}_{i,j}^k - \widetilde{X}_{i,j-1}^k)] - \delta_2\alpha\widetilde{F}_{i,j}^k h_j^{k+1} - \delta_2 f_{i,j}^{k+1} = 0;$$

$$\sum_{i=1}^{3}[(\widetilde{X}_{i,j}^k - \widetilde{X}_{i,j-1}^k)(x_{i,j}^{k+1} - x_{i,j-1}^{k+1})] - 2\alpha\,\Delta_{S0}^2 h_j^{k+1} = 0;$$

$$s_j^{k+1} - s_{j-1}^{k+1} - \Delta_{S0}\alpha h_j^{k+1} = 0 \qquad (i=1...3,\quad j=1..N,\quad k=1...M),$$

where δ_1, δ_2 are the equations constants equal to:

$$\delta_1 = \frac{\Delta_t^{\ 2}}{\lambda\Delta_{S0}^2}\ ;\ \delta_2 = \frac{\Delta_t^2}{\lambda}.$$

$\widetilde{X}_{i,j}^k$ is the approximation of the coordinate *i (i=1...3)* at the cable point *j (j=1...N)* during the time step *k (k=1...M),* $\widetilde{T}_j^k$ approximates the cable tension at point j during the time step, k $x_{i,j}^{k+1}$, h_j^{k+1} approximate the increments of the coordinates and the tension respectively, $\widetilde{F}_i^k$, f_i^{k+1} are the approximations of the projections on axis i of the external forces and their increments respectively

For the time step $k+1$ and for each integer j the Eqs. (9) are a set of algebraic linear equations relative to 13 unknown quantities $x_{i,j-1}^{k+1}, x_{i,j}^{k+1}, x_{i,j+1}^{k+1}, h_{j-1}^{k+1}, h_j^{k+1}, s_{j-1}^{k+1}, s_j^{k+1}$. Each of these sets connects the quantities at three cable points $(j-1), j, (j+1)$ for three time steps $(k-1), k, (k+1)$.

Thus, we have a system of algebraic equations for arbitrary step of time (*k*+1):

$$A_{1,j}\; x_{i,j-1}^{k+1} + A_{2,j}\; x_{i,j}^{k+1} + A_{3,j}\; x_{i,j+1}^{k+1} + A_{4,i,j}\; h_{j-1}^{k+1} + A_{5,i,j}\; h_{j}^{k+1} = B_{i,j}\,;$$

$$\sum_{i=1}^{3}(C_{2i-1,j}\, x_{i,j-1}^{k+1} + C_{2i,j}\; x_{i,j}^{k+1}) - \alpha\Delta_{S0}^{2}\, h_{j}^{k+1} = 0\,; \qquad (12)$$

$$s_{j}^{k+1} - s_{j-1}^{k+1} - \alpha\,\Delta_{S0}\; h_{j}^{k+1} = 0\,,$$

where $A_{1,j}$, $A_{2,j}$, $A_{3,j}$, $A_{4,i,j}$, $A_{5,i,j}$, $B_{i,j}$, $C_{1,j}$, $C_{2,j}$, $C_{3,j}$, $C_{4,j}$, $C_{5,j}$, $C_{6,j}$ are the constant of the system of the Eqs.(12), which are equal to:

$$A_{1,j} = \delta_1 \widetilde{T}_{j-1}^{k}\,; \qquad A_{2,j} = -\delta_1(\widetilde{T}_{j}^{k} + \widetilde{T}_{j-1}^{k}) - 1\,;$$

$$A_{3,j} = \delta_1 \widetilde{T}_{j}^{k}\,; \qquad A_{4,i,j} = -\delta_1(\widetilde{X}_{i,j}^{k} - \widetilde{X}_{i,j-1}^{k})\,;$$

$$A_{5,i,j} = \delta_1(\widetilde{X}_{i,j+1}^{k} - \widetilde{X}_{i,j}^{k}) + \delta_2\,\alpha\,\widetilde{F}_{i,j}^{k}\,; \qquad B_{i,j} = x_{i,j}^{k-1} - 2x_{i,j}^{k} - \delta_2\, f_{i,j}^{k}\,; \qquad (13)$$

$$C_{2i-1,j} = -C_{2i,j} = \widetilde{X}_{i,j-1}^{k} - \widetilde{X}_{i,j}^{k}\,.$$

The matrix of the coefficients of the Eqs.(12) is a band one, positively defined so that the solution of this system may be carried out while using standard programs. The assumption of the positive tension is valid at every cable point and it is necessary to ensure nonsingularity of the matrix. The system of the Eqs.(12) is solved for each time step so that the results from the preceding step are used as initial conditions for the next step.

The Eqs.(12) are similar to the ones in [6, 7], but the tension and the cable length there are assumed as constants during each time step and the system is solved only for unknown quantities $x_{i,j-1}^{k+1}$, $x_{i,j}^{k+1}$, $x_{i,j+1}^{k+1}$. In [6, 7] the increments of the tension and of the cable length are calculated only after the current time step leaving out of account the cable elasticity (the dependence between the cable tension and its length) for the current time step. This procedure causes an accumulation of calculation errors growing enough quickly in step for low tension. Moreover, such calculations are quite impossible for nontensible cable. After a lapse of 0.5...0.6 sec the error is measured in tens of percents. This phenomena is most pronounced for slightly stretched cables, when the maximum of tension (the second one) is achieved after a relatively prolonged time.

The Eqs. (12) are free from the errors described above.

5. Results of calculations and discussion.

Two examples of calculations are made for the typical cables (its parameters are listed in table 1). The cables are assumed to be driven by the distributed electromagnetic transverse dropped force, described to Eqn. (9).

The plots of the cable displacement of the midpoint of the span and the maximum cable tension versus time are presented for two examples of calculations in fig. 1-4. Every figure contains the results corresponding to the two methods of calculation: the method described in [6,7] (dotted lines) and the present one (continuous lines). The results of the computation according to [6, 7] were carried out and put at the author's disposal by the courtesy of Dr. A. Polevoy.

The results of the calculations for a relatively strongly stretched and light cable under relatively high short-circuit current are shown in fig. 1 (the tension) and in fig. 2 (the trajectory of the midpoint). The cable trajectory is a “rotational” one for the all cable points. An accumulation of calculation’s errors is detectable after the time of 0.4 sec.

The results of the calculations for a relatively slightly stretched and heavy cable under relatively low short-circuit current are presented in fig. 3 (the tension) and in fig. 4 (the trajectory of the midpoint).

Table 1. Cable parameters

Parameter	For figs. 1, 2	For figs. 3, 4
Mass per unit length, *m*	1.23, kg/m	2.56, kg/m
Cable length, ℓ	37, m	66, m
Stretch cable tension	7550, N	6030, N
Cross section area, As	$2*10^5$, N/mm^2	$6.9*10^4$, N/mm^2
Modulus of elasticity, *E*	$2*10^5$, N/mm^2	$6.9*10^4$, N/mm^2
Duration of fault	0.3, sec	0.3, sec
Initial electromagnetic force per unit length	3*mg,* N	2*mg,* N

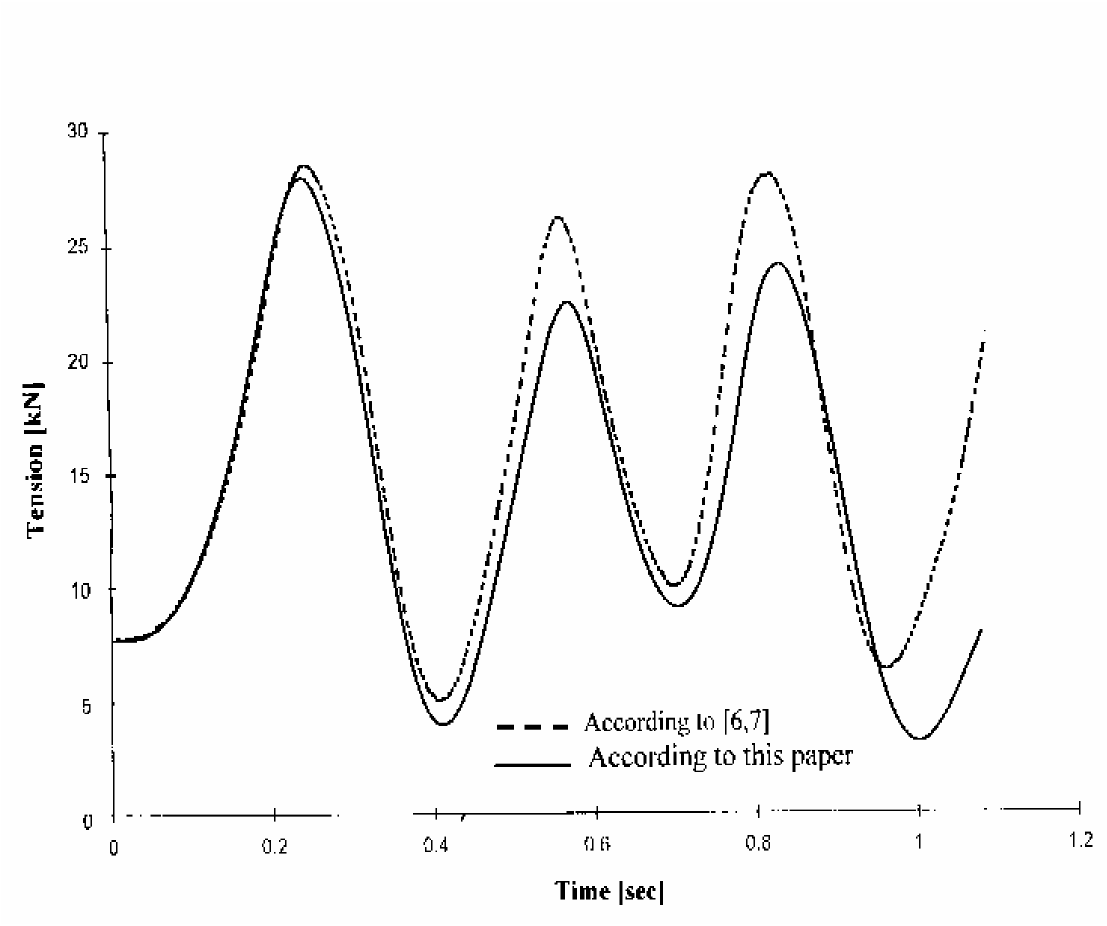

Fig. 1 Trajectories of midpoint's movement for strongly stretched and light cable.

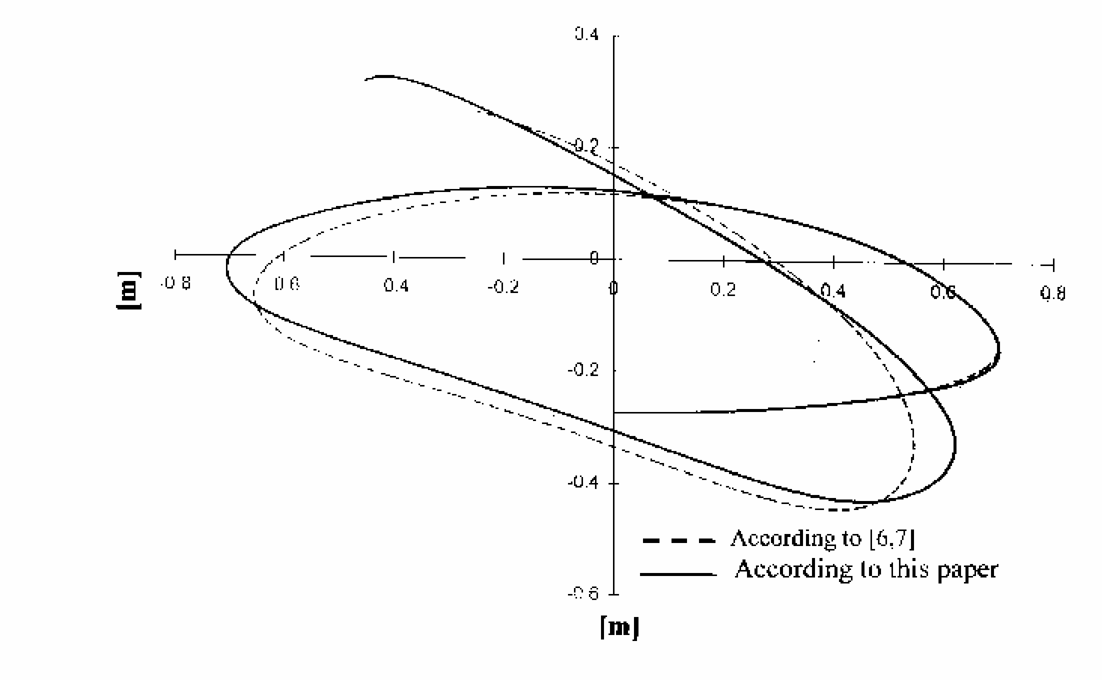

Fig. 2 Tension for strongly stretched and light cable.

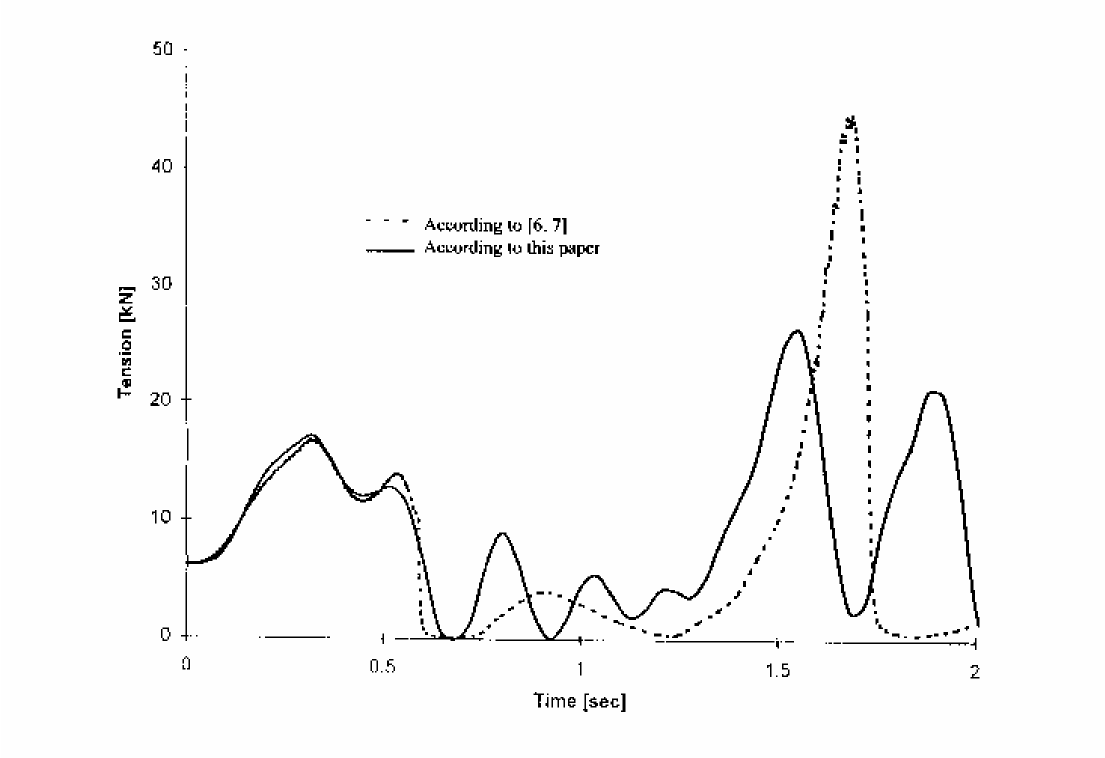

Fig. 3 Trajectories of midpoint's movement for slightly stretched and heavy cable.

Two kind of cable movement can be distinguished in fig. 4. The "oscillation" movement shown in continuous line and the movement with cable drop after its deviation presented in dotted line. The accumulation of calculation's errors is typical of last kind of movement. The second maximum of the tension in fig. 3 corresponds to the time of cable drop. After the time of 0.6 sec and the calculation's error reaches up to 80%.

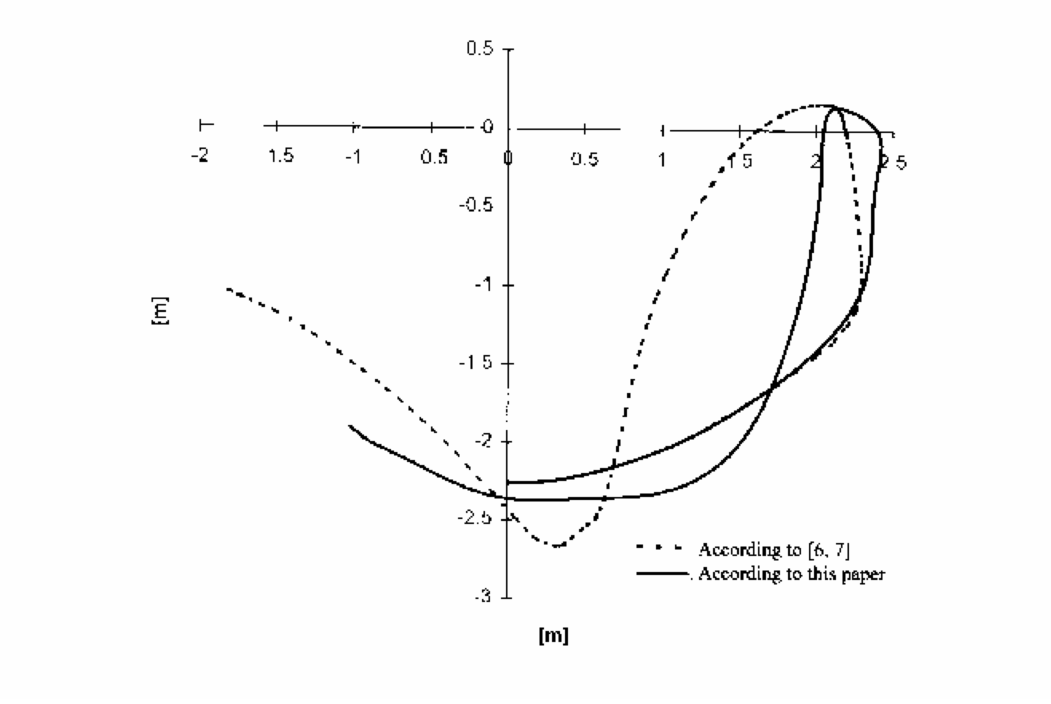

Fig. 4 Tension for strongly slightly stretched and heavy cable.

6. Probability approach.

The flow diagram of the probabilistic calculations is shown in fig. 5. A typical plot of the probability density $p_I(I_a)$ of the distribution of the expected amplitude I_a of the short-circuit current (faults) as a result of Monte-Carlo simulations [3] is presented in fig. 6. The deterministic model of the cable dynamics enables us to build the function of the maximum cable tension $T_{\max}$ during the fault versus the amplitude of the short-circuit current (fig. 7). This function usually is monotone for stretched cables and that enable us to simplify the process of the calculations.

The probability density $p_T(T_{\max})$ of the maximum cable tension for the fault may be calculated by using the formula:

$$p_T(T_{\max}) = |\frac{dT_{\max}}{dI_k}|\, p_I(I_a) \tag{14}$$

When knowing $p_T(T_{\max})$, the probability of the cable tension $F_T(T_s) = \{P(T_{\max} > T_s)\}$ which is greater than some specified value T_s may be calculated.

The mean quantity of the faults N during one year per 100 km of electrical transmission lines system is known usually from statistical data. The mean number of the faults N_f during the number of years of expected service life N_Y is calculated for the electrical transmission lines system in length L:

$$N_f = N_Y N L \tag{15}$$

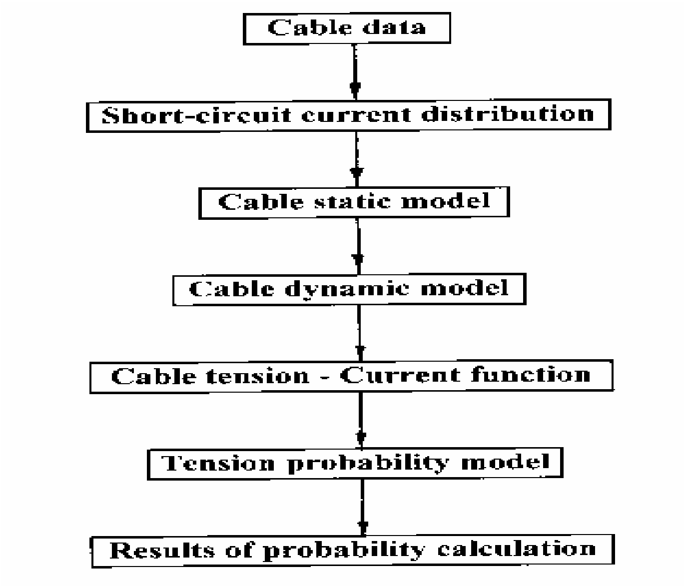

Fig. 5. The flow diagram of the probabilistic calculations.

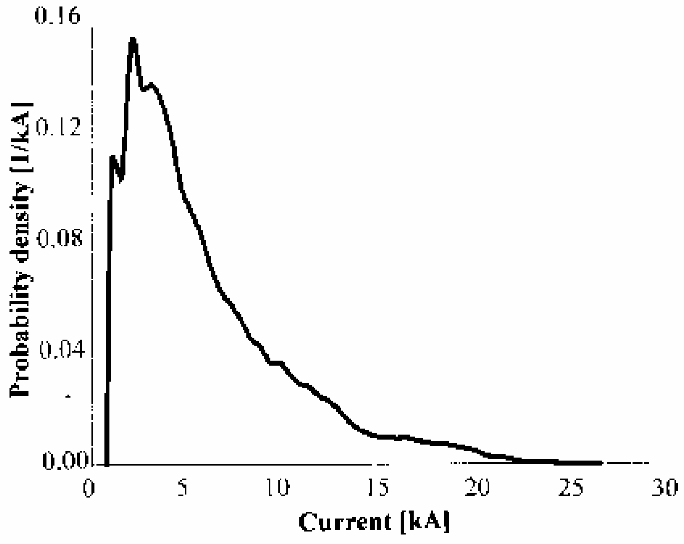

Fig. 6 The distribution of the short-circuit current.

The probability $F_{TY}(T_s)$ of the appearance of the cable tension, which is greater than some specified value T_s at least once during the expected service life, is calculated as:

$$F_{TY}(T_s) = 1 - [1 - F_T(T)]^{N_f} \tag{16}$$

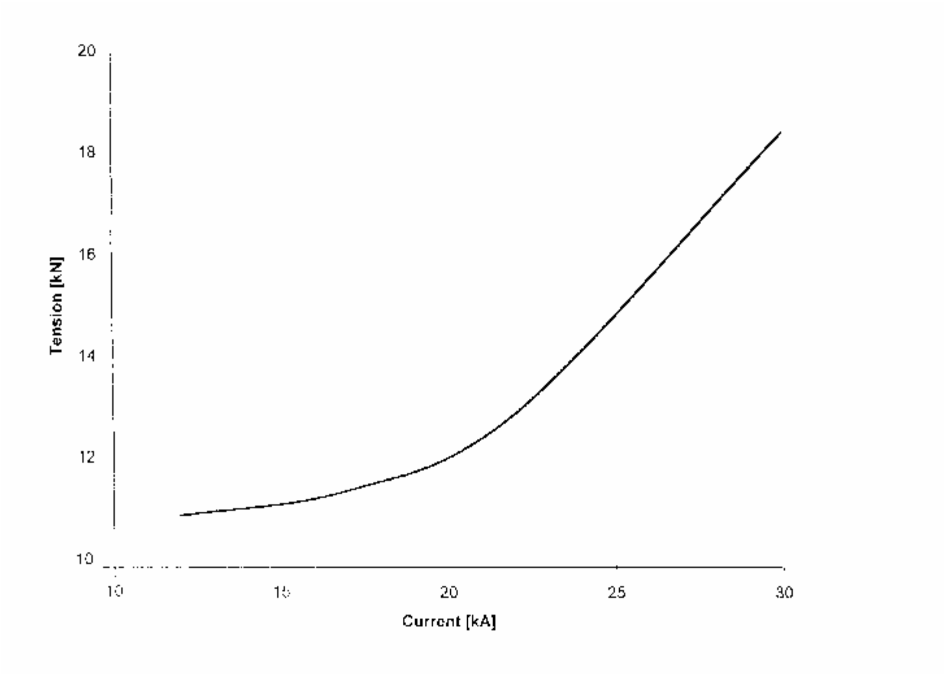

Fig. 7 Maximum cable tension versus the amplitude of the short-circuit current.

A typical plot of such probability is shown in fig. 8. The plot tends to the value of 1.0 almost without exception along the axis of the tension and it is different from 1.0 only for currents close to the maximum ones.

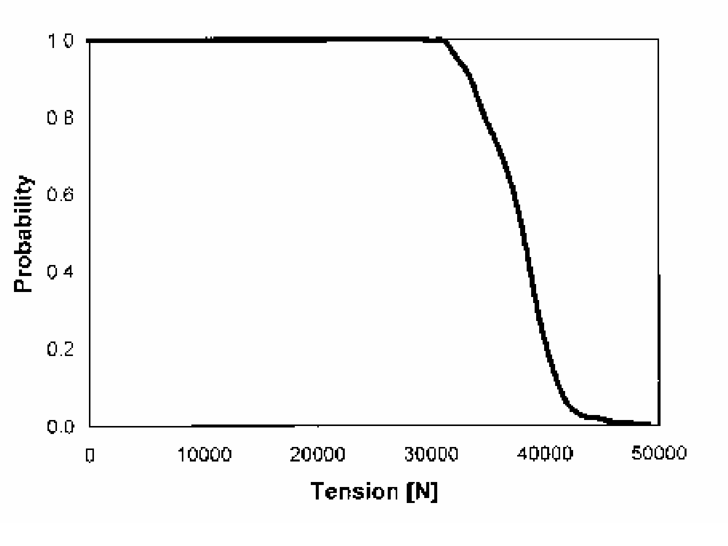

Fig. 8 The probability of the short-circuit current during the service life.

The mean time (rate) t_m between the appearance of the events for which the tension is greater than some specified value T_s during the expected service life is calculated as:

$$t_m = \frac{1}{NLF_{TY}(T_s)} \tag{17}$$

The plot of the mean time between the appearance of these events is presented in fig. 9. According to this plot the design value of maximum tension may be established for return period of 50, 100, 150 etc years. Such approach is a common practice for design of electrical transmission lines under wind loads.

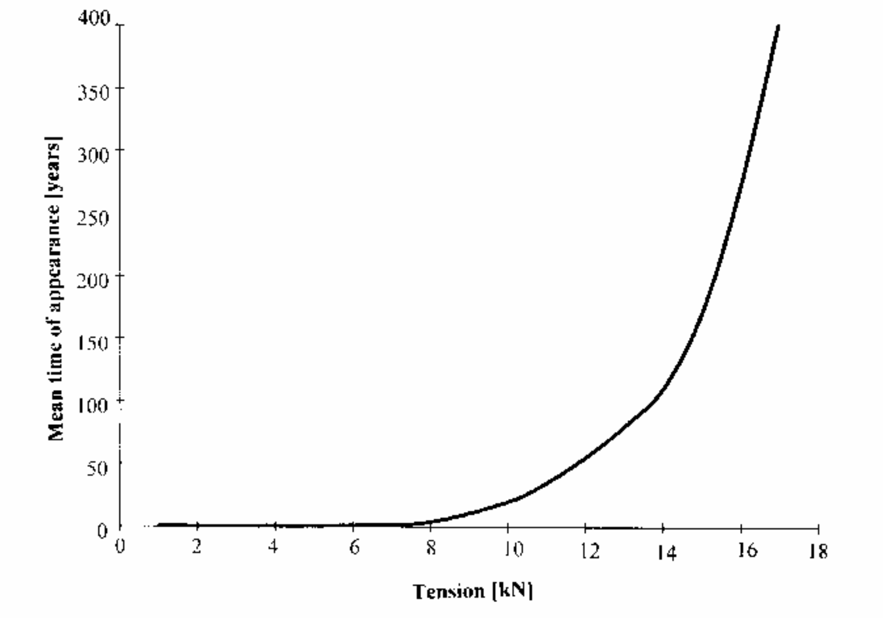

Fig. 9 The mean time between the appearance of the tension.

7. Conclusion.

The refined method for calculation of cables dynamics enables us to reduce the cumulative computational errors, to obviate the difficulties of the computation for slightly stretched cables and to solve the dynamics problem of untensible cables.

The criterion for the probabilistic approach to design of the electrical substations, which is similar to that used for random wind load, is presented in the paper.

8. Acknowledgment.

The Israel Electric Corporation Ltd. supported this research. The author wishes to acknowledge Dr. Alex Polevoy, Dr. Hanoh Ben Haim and Dr. Gregory Levitin for their attention in the discussion of the paper results.

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